Nonextensive analysis on the local structure entropy of complex networks

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Abstract

The local structure entropy is a new method which is proposed to identify the influential nodes in the complex networks. In this paper a new form of the local structure entropy of the complex networks is proposed based on the Tsallis entropy. The value of the entropic index q will influence the property of the local structure entropy. When the value of q is equal to 0, the nonextensive local structure entropy is degenerated to a new form of the degree centrality. When the value of q is equal to 1, the nonextensive local structure entropy is degenerated to the existing form of the local structure entropy. We also have find a nonextensive threshold value in the nonextensive local structure entropy. When the value of q is bigger than the nonextensive threshold value, change the value of q will has no influence on the property of the local structure entropy, and different complex networks have different

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nonextensive threshold value.

The results in this paper show that the new nonextensive local structure entropy is a generalised of the local structure entropy. It is more reasonable and useful than the existing one.
Keywords: Complex networks, Local structure entropy, Tsallis entropy, Nonextensive statistical mechanics

1. Introduction

The complex networks is a model which can used to describe those complex relationship in the real system, such as the biological, social and technological systems [1, 2]. Many property of the complex networks have illuminated by these researchers in this filed, such as the network topology and dynamics [3, 4], the property of the network structure [2, 5], the self-similarity and fractal property of the complex networks[6, 7], the controllability and the synchronization of the complex networks [8, 9] and so on [10, 11, 12, 11, 6, 13]. How to identify the influential nodes in the complex networks has attracted many researchers to study it. Recently, a local structure entropy of the complex networks is proposed to identify the influential nodes in the complex networks [14]. In the local structure entropy the node's influence on the whole network is replaced by the local network. The degree entropy of the local network is used as the measure of the influence of the node on the whole network. The local structure entropy is based on the shannon entropy. In this paper the Tsalli entropy which is proposed by Tsalli et.al [15] is used to analysis the property of the local structure entropy.

Depends on the nonextensive statistical mechanics, the relationship between each node can be described by the nonextensive additivity.

In this paper, the property of the local structure entropy is analysed by the nonextensive statistical mechanics. Depends on the Tsallis entropy, a new form of the local structure entropy is proposed in this paper. In the nonextensive local structure entropy, the influences of the node on the whole network is changed by the entropic index q. The nonextensive in the local structure entropy is changed correspond to value of q and the nonextensive additivity is restricted by the value of q. We also find the nonextensive threshold value of q in the nonextensive local structure entropy. When the value of q is bigger than the nonextensive threshold value, then the property of the local structure entropy will not be controlled by the q. When the value of q is equal to 0, then the local structure entropy is degenerated to another form of the degree centrality. The nonextensive local structure entropy is a generalised of the local structure entropy.

The rest of this paper is organised as follows. Section 2 introduces some preliminaries of this work, such as the local structure entropy of complex networks and the nonextensive statistical mechanics. In section 3, the analysis of the local structure entropy based on the nonextensive is proposed. The application of the nonextensive analysis in these real networks is shown in the section 4. Conclusion is given in Section 5.

2. Preliminaries

2.1. Local structure entropy of complex networks

There many methods are proposed to identify the influential nodes in the complex networks. The degree centrality and the betweenneess centrality are the wildly used method to identify the influential nodes in the complex networks. Recently, the "Local structure entropy" of the complex networks which is based on the degree centrality and the shannon entropy is proposed [14]. The details of the local structure entropy of the complex networks is shown as follows [14].

The definition of the local structure entropy can be divided into three steps, the details are shown as follows [14].

Step 1 Creating a local network: First, choose one of the node in the network as the central node. Second, find all of the nodes in the network which are connect with the central node in directly. Third, create a local network which contains the central node and his neighbour nodes.

Step 2 Calculating the unit of the local structure entropy: Calculate the degree of each node and the total number of the degree in the local network. The unit of the local structure entropy can be represents as the p_{ij}, it is defined in the Eq.(2).

Step 3 Calculating the local structure entropy of each node: The definition of the local structure entropy for each node is shown in the Eq.(1).

The definition of the local structure entropy of the complex networks is shown as follows [14].

$$LE_i = -\sum_{j=1}^{n} p_{ij} \log p_{ij} \tag{1}$$

Where the LE_i represents the local structure entropy of the ith node in the complex networks. The n is the total number of the nodes in the local network. The p_{ij} represents the percentage of degree for the jth node in the local network. The definition of the p_{ij} is shown in the Eq.(2).

$$p_{ij} = \frac{degree(j)}{\sum\limits_{j=1}^{n} degree(j)} \tag{2}$$

An example of the process to calculate the local structure entropy is shown in the Fig.1.

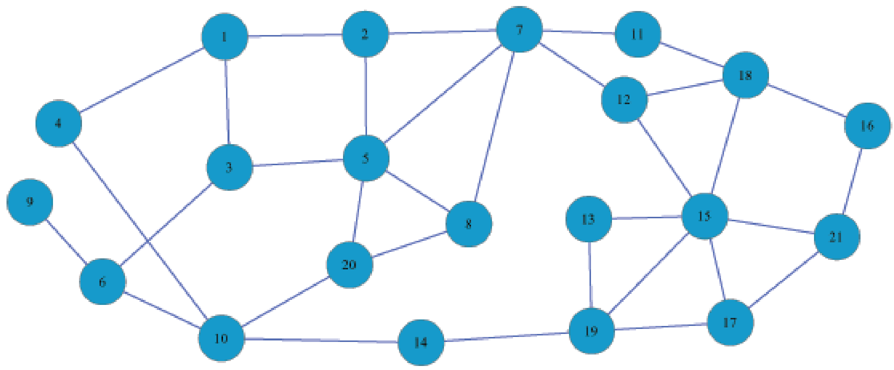
It is clear that in the local structure entropy the influence of the node on the whole network is replaced by the influence of the local network on the whole network [14].

2.2. Nonextensive statistical mechanics

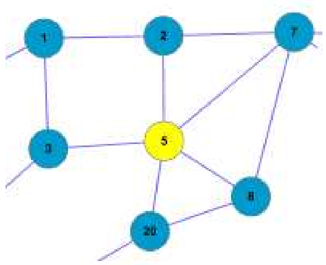
The entropy is defined by Clausius for thermodynamics [16]. For a finite discrete set of probabilities the definition of the Boltzmann-Gibbs entropy [**?**] is given as follows:

$$S_{BG} = -k \sum_{i=1}^{N} p_i \ln p_i \tag{3}$$

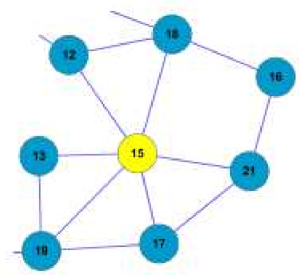
The conventional constant k is the Boltzmann universal constant for thermodynamic systems. The value of k will be taken to be unity in information theory [17].



(a) The example network A



(b) The $5th$ node.



(c) The $15th$ node

Figure 1: In the example network A, different node has different value of degree. The LE_i of each node is different to each others. We use the node 5 and node 15 to show the details of the calculation of the local structure entropy of each node. First, the local structure entropy of the node 5. The node 5 has 5 neighbours, the node 2, 3, 20, 8, 7. The degree of each neighbour node is 3, 3, 3, 3, 5. The degree of the node 5 is 5. The total degree in the local network is 19. Then the set of the degree in the local network is $D_5 = 3/19, 3/19, 3/19, 5/19, 5/19$. Then the LE_5=1.8684. Second, the local structure entropy of the node 15. The node 15 has 6 neighbours, the node 12, 13, 17, 18, 19, 21. The degree of each neighbour node is 3, 2, 3, 4, 4, 3. The degree of node 15 is 6. The total degree in the local network is 25. The set of the degree in the local network is $D_15 = 3/25, 2/25, 3/25, 4/25, 4/25, 3/25, 6/25$ Then the LE_15=1.89426. From the definition of the local structure entropy the node 15 is more influential than the node 5 in the example network.

In 1988, a generalised entropy have been proposed by Tsallis [15]. It is shown as follows:

$$S_q = -k\sum_{i=1}^{N} p_i \ln_q \frac{1}{p_i} \tag{4}$$

The $q - logarithmic$ function in the Eq. (4) is presented as follows:

$$\ln_q p_i = \frac{{p_i}^{1-q} - 1}{1-q}(p_i > 0; q \in \Re; ln_1 p_i = lnp_i) \tag{5}$$

Based on the Eq. (5), the Eq. (4) can be rewritten as follows:

$$S_q = -k\sum_{i=1}^{N} p_i \frac{{p_i}^{q-1} - 1}{1-q} \tag{6}$$

$$S_q = -k\sum_{i=1}^{N} \frac{{p_i}^{q} - p_i}{1-q} \tag{7}$$

$$S_q = k\frac{1 - \sum\limits_{i=1}^{N} {p_i}^{q}}{q-1} \tag{8}$$

Where N is the number of the subsystems.

Based on the Tsallis entropy, the nonextensive theory is established by Tsallis et.al. The nonextensive statistical mechanics is a generalised statistical mechanics.

3. Nonextensive analysis of the local structure entropy of complex networks

The main idea of the local structure entropy is try to use the influence of the local network to replace the influence of the node on the whole network

[14]. However, in the definition of the local structure entropy of each node, the relationship between each node in the local network is extensive. In order to illuminate the property of the local structure entropy, in this paper the nonextensive statistical mechanic is used in the definition of the local structure entropy.

Depends on the Tsallis entropy, the definition of the local structure entropy is redefined as follows:

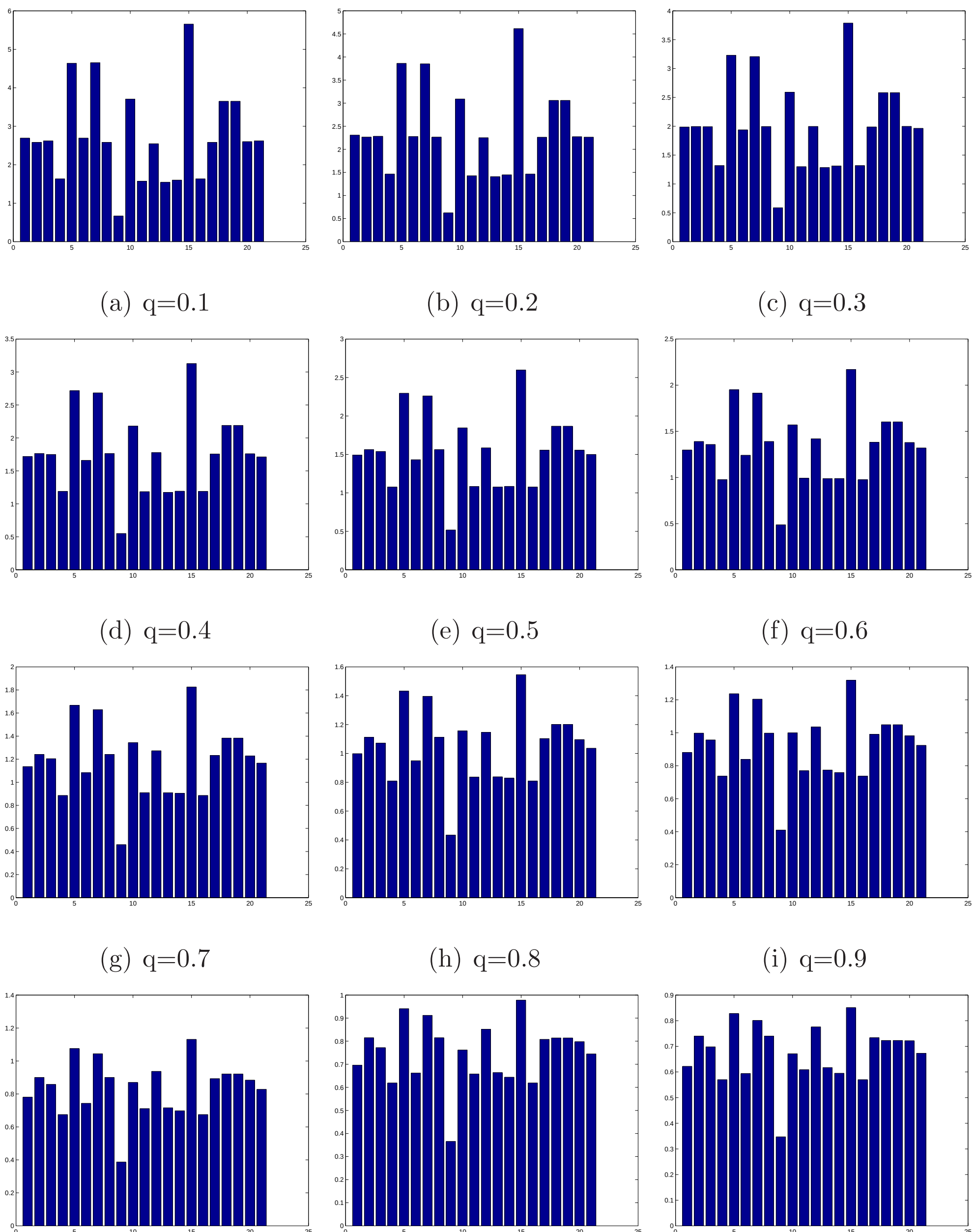
$$S_{q_i} = -k \sum_{i=1}^{N} p_i \ln_q \frac{1}{p_i} \tag{9}$$

Where in the Eq.(9), the logarithmic function in the local structure entropy is replaced by the $q-logarithmic$ function in the Eq.(5). The S_{q_i} is the new local structure entropy of the node i. It is defined based on the Tsallis entropy [18]. The p_{i_j} is defined in the Eq.(2). The q is the nonextensive entropic index.

We use the example network A which is shown in the Fig.1 to show the nonextensive property of the local structure entropy.

The order of the influential nodes in the example network A is shown in the Table 2.

The order of the influential nodes in the example network A is shown in the Table 4. Where in the Table 4, the D_{order} represents the order of the influential nodes in the example network A, the D_{order1} represents another order of the influential nodes in the example network A. Both of the D_{order} and the D_{order1} is based on the degree of each node. The $LE_{order} q = x$ represents



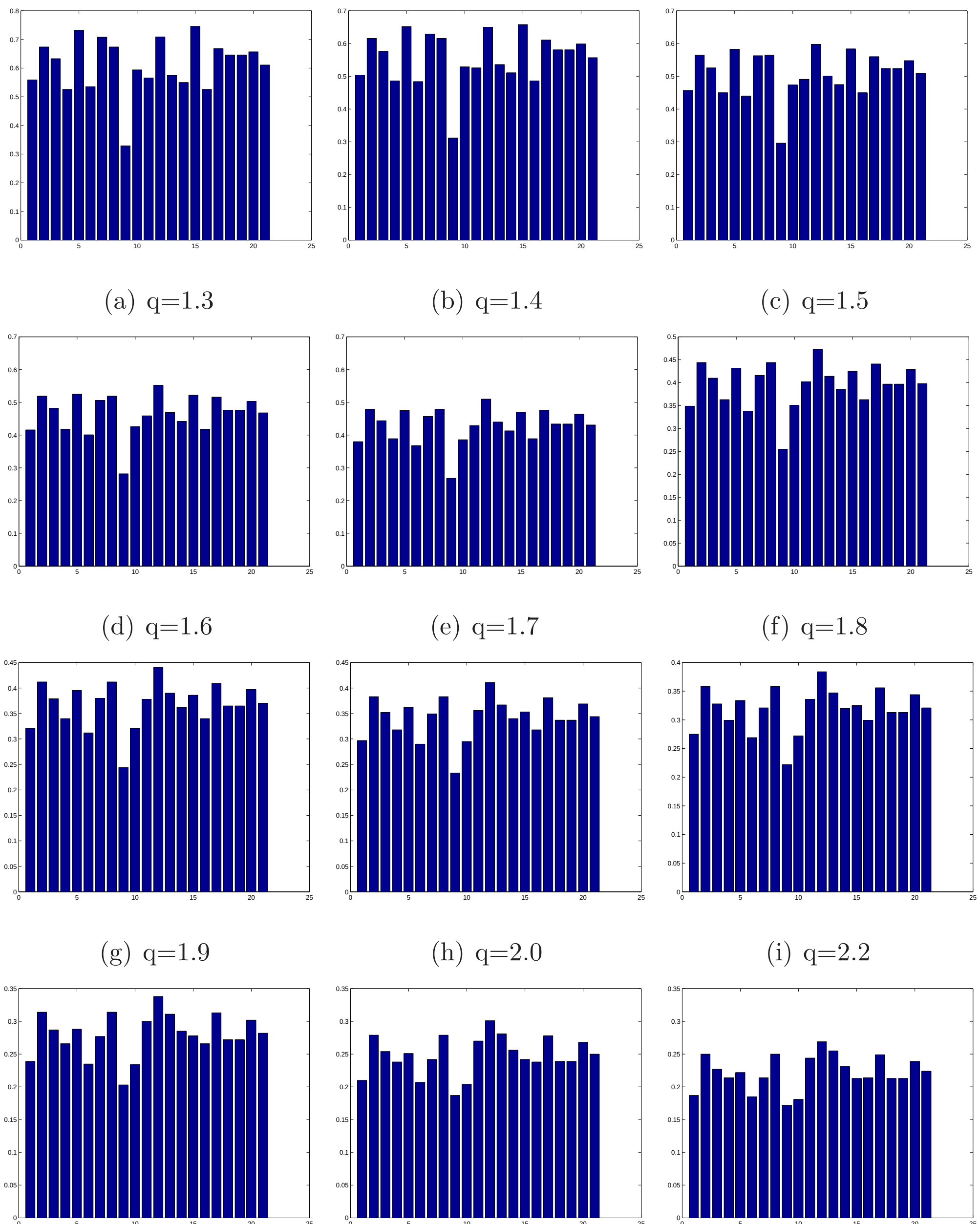
(a) q=0.1 (b) q=0.2 (c) q=0.3

(d) q=0.4 (e) q=0.5 (f) q=0.6

(g) q=0.7 (h) q=0.8 (i) q=0.9

(j) q=1.0 (k) q=1.1 (l) q=1.2

Figure 2: The figure show the value of the nonextensive local structure entropy of each node in the example network A. The value of q is big than 0.1 and small than 1.2. The caption of the subfigure show the value of q. The Abscissa in those subfigure represents the node's number and the ordinate represents the value of nonextensive local structure entropy.



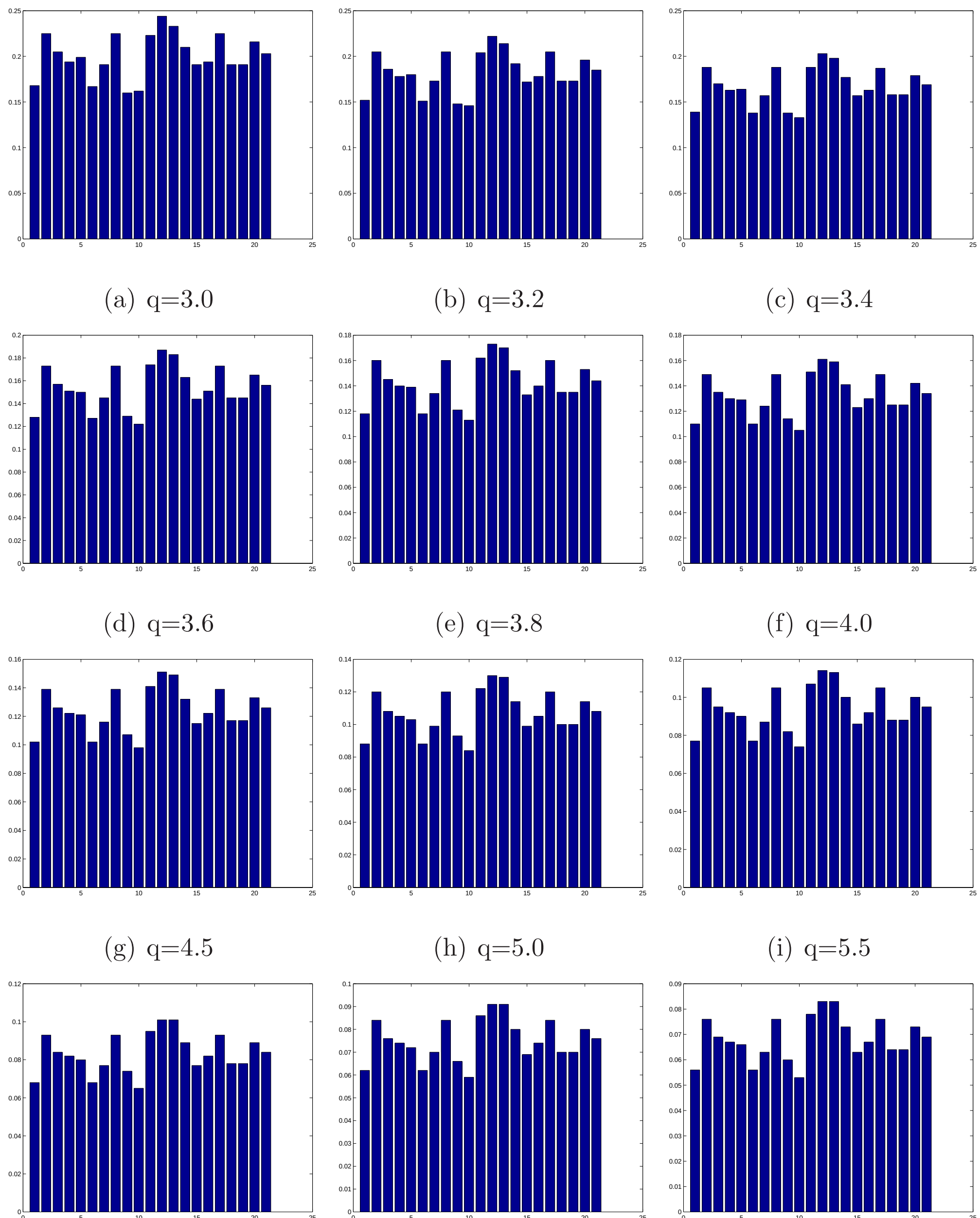
(a) q=1.3 (b) q=1.4 (c) q=1.5

(d) q=1.6 (e) q=1.7 (f) q=1.8

(g) q=1.9 (h) q=2.0 (i) q=2.2

(j) q=2.4 (k) q=2.6 (l) q=2.8

Figure 3: The figure show the value of the nonextensive local structure entropy of each node in the example network A. The value of q is big than 1.3 and small than 2.8. The caption of the subfigure show the value of q. The Abscissa in those subfigure represents the node's number and the ordinate represents the value of nonextensive local structure entropy.



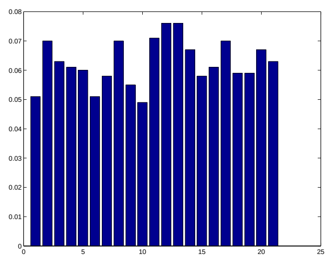
(a) q=3.0 (b) q=3.2 (c) q=3.4

(d) q=3.6 (e) q=3.8 (f) q=4.0

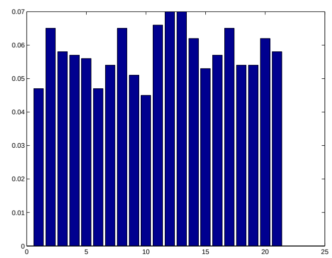
(g) q=4.5 (h) q=5.0 (i) q=5.5

(j) q=6.0 (k) q=6.5 (l) q=7.0

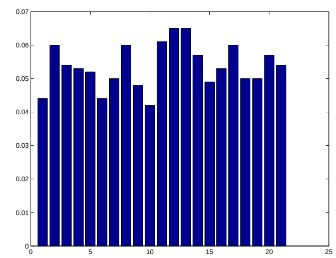
Figure 4: The figure show the value of the nonextensive local structure entropy of each node in the example network A. The value of q is big than 3.0 and small than 7.0. The caption of the subfigure show the value of q. The Abscissa in those subfigure represents the node's number and the ordinate represents the value of nonextensive local structure entropy.



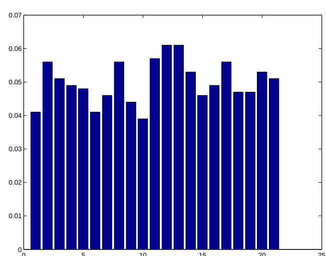
(a) q=7.5



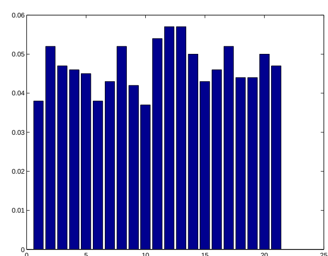
(b) q=8.0



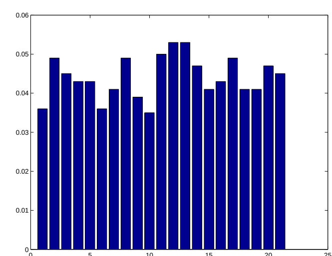
(c) q=8.5



(d) q=9.0



(e) q=9.5



(f) q=10.0

Figure 5: The figure show the value of the nonextensive local structure entropy of each node in the example network A. The value of q is big than 7.5 and small than 10. The caption of the subfigure show the value of q. The Abscissa in those subfigure represents the node's number and the ordinate represents the value of nonextensive local structure entropy.

Table 1: The order of the influential nodes in the example network A with the change of the value of q

Node order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
q=0	15	7	5	10	19	18	6	1	21	3	20	17	8	2	12	16	4	14	11	13	9
q=0.1	15	5	7	10	18	19	1	3	6	20	8	2	21	17	12	16	4	14	11	13	9
q=0.2	15	5	7	10	19	18	20	12	8	2	3	17	1	21	6	16	4	14	11	13	9
q=0.3	15	5	7	19	18	10	12	8	2	20	17	3	1	21	6	14	16	4	11	13	9
q=0.4	15	5	7	19	18	10	12	8	2	20	17	3	21	1	6	14	11	13	16	4	9
q=0.5	15	5	7	19	18	10	12	8	2	17	20	3	21	1	6	11	14	13	16	4	9
q=0.6	15	5	7	19	18	10	12	8	2	17	20	3	21	1	6	11	13	14	16	4	9
q=0.7	15	5	7	19	18	10	12	8	2	17	20	3	21	1	6	13	11	14	16	4	9
q=0.8	15	5	7	19	18	12	10	8	2	17	20	3	21	1	6	13	11	14	16	4	9
q=0.9	15	5	7	12	19	18	8	2	17	20	10	3	21	1	6	13	11	14	16	4	9
q=1	15	5	7	12	8	2	19	18	17	20	3	10	21	1	13	6	11	14	16	4	9
q=1.1	15	5	7	12	8	2	17	19	18	20	3	21	10	1	13	11	14	6	16	4	9
q=1.2	15	5	12	7	8	2	17	20	19	18	3	21	10	13	11	1	14	6	16	4	9
q=1.3	15	5	12	7	8	2	17	20	19	18	3	21	13	10	11	14	1	16	4	6	9
q=1.4	12	15	5	8	2	7	17	20	3	19	18	21	13	11	14	10	1	16	4	6	9
q=1.5	12	5	15	8	2	17	7	20	3	19	18	13	21	11	14	10	16	4	1	6	9
q=1.6	12	8	2	17	5	15	20	7	3	13	19	18	21	11	14	16	4	10	1	6	9
q=1.7	12	8	2	17	5	20	15	7	13	3	11	21	18	19	14	16	4	10	1	6	9
q=1.8	12	8	2	17	20	5	13	15	7	3	11	21	18	19	14	16	4	1	10	6	9
q=1.9	12	8	2	17	20	13	5	11	15	3	7	21	14	19	18	16	4	1	10	6	9
q=2	12	8	2	17	13	20	11	5	3	15	7	21	14	19	18	16	4	1	10	6	9
q=2.2	12	8	2	17	13	20	11	5	3	14	21	15	7	19	18	16	4	1	6	10	9
q=2.4	12	13	8	2	17	11	20	14	3	5	21	15	7	18	19	16	4	1	6	10	9
q=2.6	12	13	8	2	17	11	20	14	3	21	5	16	4	7	15	19	18	1	6	10	9
q=2.8	12	13	2	8	17	11	20	14	3	21	5	16	4	7	19	18	15	1	6	10	9
q=3	12	13	8	2	17	11	20	14	3	21	5	16	4	19	18	7	15	1	6	9	10
q=3.2	12	13	11	8	2	17	20	14	3	21	5	16	4	18	19	7	15	1	6	9	10
q=3.4	12	13	11	2	8	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=3.6	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=3.8	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=4	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=4.5	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=5	12	13	11	8	2	17	20	14	3	21	16	4	5	18	19	7	15	9	1	6	10
q=5.5	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=6	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=6.5	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=7	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=7.5	12	13	11	8	2	17	20	14	3	21	16	4	5	18	19	7	15	9	1	6	10
q=8	12	13	11	8	2	17	20	14	3	21	16	4	5	18	19	7	15	9	1	6	10
q=8.5	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=9	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=9.5	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10
q=10	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10

Table 2: The degree of each node in the example network A

Node number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Degree	3	3	3	2	5	3	5	3	1	4	2	3	2	2	6	2	3	4	4	3	3

the order of the influential nodes which is identified by the nonextensive local structure entropy with different value of q. The LE_{order} represents the order of the influential nodes which is identified by the local structure entropy.

Table 3: The order of the influential nodes in the example network A

D_{order}	15	5	7	10	18	19	1	2	3	6	8	12	17	20	21	4	11	13	14	16	9
D_{order1}	15	7	5	10	19	18	6	1	21	3	20	17	8	2	12	16	4	14	11	13	9
Degree	6	5	5	4	4	4	3	3	3	3	3	3	3	3	3	2	2	2	2	2	1
$LE_{order} q = 0$	15	7	5	10	19	18	6	1	21	3	20	17	8	2	12	16	4	14	11	13	9
$LE_{order} q = 1$	15	5	7	12	8	2	19	18	17	20	3	10	21	1	13	6	11	14	16	4	9
LE_{order}	15	5	7	12	8	2	19	18	17	20	3	10	21	1	13	6	11	14	16	4	9

The results in of the test of the local structure entropy based on the nonextensive statistical have show the influence of the nonextensive additivity between each node on the local structure entropy.

When the value of the entropic index q is equal to 0. Then the value of the local structure entropy on each nodes is corresponded to the number of the degree of each node. The influence of the local network is degenerated to the degree's influence on the whole network. Therefore, the order of the influential nodes which is identified by the local structure entropy in the example network A is the same as the the order based on the degree value. In other word, when the value of q is equal to 0, then influence of the components on the local structure entropy is equal to others. The value of the

local structure entropy for each node is decided by the node's degree. When the value of q is equal to 1, then the additivity among there components of local structure entropy is based on the degree of the node in the local network. When the value of q is bigger than 3.6, the order of the influential nodes in the example networks is stable. It means that when the value of q is bigger than 3.6, the nonextensive additivity among those components is stable. Change the value of q has no influence on the order of the local structure entropy. The 3.6 is a threshold value of the nonextensive in the local structure entropy of example network A. The P_{value} is used to represents the threshold value of the nonextensive in the local structure entropy.

The details can be illuminated in six parts:

Case 1 When **q=0**, the relationship between the components in the local structure entropy is equal to each others. The value of the local structure entropy is decided by the number of the components in it. In the local structure entropy which is based on the degree distribution, the order of the influential nodes in the network is equal to the order which is identified by the degree centrality.

Case 2 When **0<q<1**, these components which have small value are the main part of the local structure entropy.

Case 3 When **q=1**, the nonextensive additivity in the local structure entropy is degenerated to the extensive additivity. The nonextensive local structure entropy degenerate to the local structure entropy.

Case 4 When $\mathbf{1}<\mathbf{q}<P_{value}$, the components which have big value are the main part of the local structure entropy.

Case 5 When $\mathbf{q}=P_{value}$, the nonextensive additivity in the local structure entropy achieve to a stable state.

Case 6 When $\mathbf{q}>P_{value}$, the order of the influential nodes in the complex networks is achieve to stable. Change the value of q will have no influence on the order of the influential nodes in the complex networks.

The order of the influential nodes in the complex can be described in three state: 1. The initial state $Order_{q=0}$. When the value of q is equal to 0, the order of the influential nodes in the complex networks. This state is the same as the order which is identified by the degree centrality. 2. The local structure entropy state $Order_{q=1}$. When the value of q is equal to 1, the order of the influential nodes in the complex networks. The order is decided by the local structure entropy. 3. The stable state $Order_{stable}$. When the value of q is bigger than the nonextensive threshold value, the order of the influential nodes in the complex networks is stable.

The three state of the example network A is shown as follows:

Table 4: The three state of the influential nodes in the example network A

Node order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
$Order_{q=0}$	15	5	7	10	18	19	1	2	3	6	8	12	17	20	21	4	11	13	14	16	9
$Order_{q=1}$	15	5	7	12	8	2	19	18	17	20	3	10	21	1	13	6	11	14	16	4	9
$Order_{stable}$	12	13	11	8	2	17	20	14	3	21	16	4	5	19	18	7	15	9	1	6	10

The results show that the nonextensive local structure entropy is more useful and more reasonable than the local structure entropy. The nonextensive local structure entropy is a generalised method to identify the influential nodes in the complex networks.

4. Application

In this section four real networks is use to analysis the nonextensive in the local structure entropy of it. The four networks are the Zachary's Karate Club network (Karate) [19], the US-airport network (Us-airport) [20], Email networks (Email) [20]and the Germany highway networks (Highway) [21].

The results of the nonextensive threshold value (P_{value}) of each network and the three state of the four networks is shown as follows:

Table 5: The three states of the (Karate)network

	Top 10 influential nodes										Low 10 influential nodes									
$Order_{q=0}$	34	1	33	3	2	4	32	24	14	9	13	18	22	10	15	16	19	21	23	12
$Order_{q=1}$	14	9	3	32	31	8	4	1	33	2	6	7	5	11	13	26	27	25	17	12
$Order_{stable}$	12	15	16	19	21	23	20	10	18	22	3	6	7	2	33	26	25	1	17	34

Table 6: The three states of the (Us-airport) network

	Top 10 influential nodes										Low 10 influential nodes									
$Order_{q=0}$	118	261	255	152	182	230	166	67	112	201	277	278	279	280	282	291	294	304	88	114
$Order_{q=1}$	159	292	172	131	94	109	307	301	310	305	188	31	22	171	328	330	32	122	332	327
$Order_{stable}$	88	114	241	247	257	268	277	278	279	280	22	31	13	171	328	330	122	332	32	327

The threshold value of the nonextensive (P_{value}) in the four real networks is shown in the Table 9. It is clear that the nonextensive in the local structure

Table 7: The three states of the (Email) network

	Top 10 influential nodes										Low 10 influential nodes									
$Order_{q=0}$	105	333	23	16	42	41	196	233	21	76	791	955	956	959	294	253	261	779	374	644
$Order_{q=1}$	105	3	39	16	42	54	210	390	50	332	1091	1099	1118	1119	1121	1130	1052	1103	1125	1133
$Order_{stable}$	644	236	374	779	247	394	253	261	294	955	1091	1099	1118	1119	1121	1130	1052	1103	1125	1133

Table 8: The three states of the (Highway) network

	Top 10 influential nodes										Low 10 influential nodes									
$Order_{q=0}$	693	403	300	410	758	373	217	556	207	331	247	587	678	898	779	326	1031	265	787	798
$Order_{q=1}$	219	393	698	217	404	450	543	267	331	763	1154	1155	1158	1160	1162	1163	1165	1166	1167	1168
$Order_{stable}$	265	787	798	326	1031	779	687	709	161	247	1128	1129	1131	1133	1138	1144	1145	1156	1159	1164

Table 9: The nonextensive threshold value of the four networks

Network	Nodes	edages	P_{value}
Karate	34	78	4.5
Us-airport	332	2126	7.6
Email	1133	10902	6.2
Highway	1168	2486	4.1

entropy if not based on the scale of the network.

5. Conclusion

The local structure entropy is a new method which is used to identify the influential nodes in the complex networks. In this paper, the local structure entropy of the complex networks is redefined by the nonextensive statistical mechanics. The results of the nonextensive analysis on the local structure entropy shown that when the entropic index q is equal to 0, the order of the influential nodes which is identified by the nonextensvie local structure entropy the same as the degree centrality. When the value of q is equal to 1, then the nonextensive local structure entropy is degenerated to the traditional local structure entropy. When the value of q is bigger than the nonextensive threshold value (P_{value}) the order of the influential nodes in the complex networks is stable.

It is clear that the value of q will influence the property of the local structure entropy, but it also have a range. When the value of q is smaller than P_{value} and bigger than 1, the components with big value in the local structure entropy play an important roles in the local structure entropy. When the value of q is smaller than 1, bigger than 0 the components with small value in the local structure entropy play an important roles in the local structure entropy. When the value of q is equal to 0, then the performance of the local structure entropy is decided by the numbers of the components in the local structure entropy. The nonextensive local structure entropy is degenerates to the degree centrality.

The new form of the local structure entropy which is defined based on the Tsallis entropy is more reasonable and more useful than the existing one. It is a generalised method which can be used to identify the influential nodes in the complex networks.

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